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EARLY MATHEMATICS.

Facsimile of the Rhind Mathematical Papyrus in the British Museum. With an Introduction by E. A. Wallis Budge, M.A., Litt.D., D.Lit., F.S.A., Keeper of the Egyptian and Assyrian Antiquities. 21 Plates. (Printed by Order of the Trustees.)

THE Rhind Mathematical Papyrus, a facsimile of which the Trustees of the British Museum have just issued, together with an introduction by Dr. Wallis Budge, is the document from which we gather most of what we know of the conception and use of mathematics by the ancient Egyptians. The papyrus does not contain a systematic treatise on mathematics, nor does it attempt to deal with the subject from a scientific standpoint. It consists rather of tables and sets of worked out problems, such as would constantly require to be solved by an Egyptian master-builder, land-owner, farmer or estate-agent. In consequence of the inundation, the area of an Egyptian farmer's holding was constantly changing in extent, so that the need of some practical method of measuring area was pressing. The farmer after harvest would need some plan for estimating the storage space required for his grain; the cattle-owner and employer of labour would constantly have to face problems connected with the distribution of fodder and provisions; the builder would require some method for estimating the angle of a pyramid to be erected upon a given base. Such problems as these were of every-day occurrence, and they forced the ancient Egyptian to employ his ingenuity in solving them. How far he was successful, and to what extent he proved himself a mathematician, we can gather from the Rhind Papyrus.

The papyrus consists of a roll, now broken in two pieces, which measure 10 feet 6 inches and 6 feet 9½ inches respectively. The text is written throughout in hieratic, but its actual date is not quite certain. Dr. Budge assigns it to a period not earlier than the beginning of the eighteenth dynasty, about 1700 B.C., but adds that the actual text goes back to an older period. It was probably a copy of a papyrus written in the Hyksos period, about 2000 B.C., by a scribe Âāh-mes, who states hat he himself copied an original work of the time of Âmen-em-hāt III., a king of the twelfth dynasty, about 2300 B.C.

Before treating of the contents of the papyrus it will be well to indicate briefly the limits in their knowledge of mathematics displayed by the ancient Egyptians, whose system was not so perfectly developed as that of the old Sumerian inhabitants of Babylonia. They approached their subject from the practical and not from the theoretical side; but in spite of numerous disadvantages in their system of notation, it must be admitted that they showed great ingenuity in dealing with the mathematical problems they attacked. With regard to integers the Egyptians used a decimal system of notation, but their system was inferior to the decimal system of the Arabs; for while in the Arabic notation each power of 10 is indicated by simply adding or removing a cipher, the Egyptians had a different name and symbol for each power; thus 1, 10, 100, 1000, 10,000, and 100,000 were each expressed by a different figure. This fact, however, did not prevent them from dealing without difficulty with very high numbers. In dealing with fractions, however, the case was different; here they experienced great difficulties, for, oddly enough, the Egyptian could only express divisions of unity. Any number, in fact, could be turned into a corresponding fraction by placing before it the word *re*, *e.g.* ½, ⅓, ¼, &c. By this system they could not express a fraction with a numerator greater than 1, though there was no limit to the divisor; in fact, in one of the sections on Plate *xf.* of this papyrus, the fraction 1/5432 occurs. There is one interesting exception, however, for they were able to express ⅔, but still as a division of unity, for they represented the fraction by a sign which may be rendered 1/1½. The Egyptians must have possessed multiplication tables, and those for the lower numbers were probably committed to memory; among these must have been included a table of 1/1½ values, for they could take ⅔ of a number by a single operation, and if they wished to take ⅓ of a number they halved its 1/1½ part. Besides adding and subtracting they were able, by means of their tables, without difficulty to halve and double, and to multiply and divide by ten and five. Multiplication by other numbers, however, they performed by repeated doubling and then adding; thus to multiply seven by six an Egyptian would double 7 = 14; he would then double the 14 = 28; and he would then add 14 to 28 = 42.

This brief sketch of the system of elementary Egyptian arithmetic will serve to explain the interesting arithmetical table which occupies the first six and a half plates of the Rhind Papyrus. This table was evidently worked out to help an Egyptian in his calculations with regard to fractions; it probably would not be committed to memory, but merely used for consultation like a modern table of logarithms. These six and a half plates contain the working out of a table expressing in simple fractions, with 1 for the numerator, the ratios of 2 to the odd numbers from 3 to 99, *i.e.* the fractions 2/3, 2/5, 2/7, &c. Plate i., for instance, gives the working out of these fractions from 2/3 to 2/15, from which we get the following table of results:—

2 divided by 3 = 1/1½
2 ,, ,, 5 = 1/3 + 1/15
2 ,, ,, 7 = 1/4 + 1/28
2 ,, ,, 9 = 1/6 + 1/18
2 ,, ,, 11 = 1/6 + 1/66
2 ,, ,, 13 = 1/8 + 1/52 + 1/104
2 ,, ,, 15 = 1/10 + 1/30

That this table was not due to the fancy of one scribe, but was a recognised table of values in general use, is proved by a fragment of papyrus found at Kahun in April 1889, on which part of the same table is written, and which shows the same values as the result of the division of two by the odd numbers from 3 to 21. The use of such a table is not at first sight very obvious, for if it is necessary to express 2/5 in fractions with 1 as the numerator, 1/5 + 1/5 is a simpler solution than 1/3 + 1/15. It has been suggested, however, that the Egyptians may have used the table for reducing fractions with higher numerators to fractions of unity; thus 5/11 might of course be expressed by the Egyptian as 1/11 + 1/11 + 1/11 + 1/11 + 1/11, but by means of the table a shorter

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solution can be found, *i.e.* $5/11 = 1/11 + 2/11 + 2/11 =$ (by the table) $1/11 + 2/6 + 2/66 =$ (by their table of halves) $1/11 + 1/3 + 1/33$.

On the right half of Plate xvi. is another somewhat similar but very much shorter table in a single column, giving some subdivisions of fractions which might prove useful in calculations, the results of course being expressed in divisions of unity, the table reading—

2/3 of 2/3 = 1/3 + 1/9
1/3 ,, 2/3 = 1/6 + 1/18
2/3 ,, 1/3 = 1/6 + 1/18
2/3 ,, 1/6 = 1/12 + 1/36
2/3 ,, 1/2 = 1/3

and so on. To the left of the table is a rule for finding 2/3 of a fraction, which reads as follows—

> "To take 2/3 of a fraction. When it is said to thee, 'What is 2/3 of 1/5?' make thou its double and its six times. That is its 2/3. Thus is it to be done for every fraction that occurs."

The scribe evidently meant

$$2/3 \times 1/5 = 1/5 \times 2 + 1/5 \times 6 = 1/10 + 1/30$$

This general rule is exceedingly interesting, as it is the only one that occurs on the papyrus. In fact, the rest of the purely arithmetical part of the papyrus (Plates vii.-xii.) is thrown into the form of problems which are worked out with the object of being of practical use.

We have not space to do more than indicate the principal contents of the rest of the papyrus. Parts ii. and iii. (Plates xii.-xiv.) deal with the measurements of volume and area, in which given problems are worked out in estimating the amount of grain that can be stored in cylindrical and rectangular spaces of given proportions; in others the scribe has worked out the superficial area of fields of various shapes, the linear measurements of which are given; while diagrams are in some cases drawn of the shape of the fields. In this section of the work Dr. Budge remarks that the scribe has made a good many mistakes. Part iv. (Plate xv.) deals with the measurements of pyramids, of which six examples, with five figures, are given. The first on the page has the angle of the lower half of the southern stone pyramid at Dahshûr, four have the same angle as that of the second pyramid at Gîzeh, while the angle of the last is that of the pyramid at Mêdûm. Part v. (Plates xvi.-xx.) gives a worked out series of practical problems dealing with the reckoning of farm produce, the division of food among workmen, the cost of food for birds and animals, &c. They are all questions of practical interest, worked out, more or less, by rule of thumb. As Dr. Budge remarks in his Introduction to the Papyrus:—

> "None of the examples or problems indicate that the Egyptians had any deep theoretical knowledge of arithmetic or geometry, but all of them show that they were very ready in making practical calculations."

Since the late Dr. Samuel Birch published an account of the contents of this document in the *Zeitschrift für Aegyptische Sprache* in 1868, the Rhind Papyrus has formed the subject of much discussion among both mathematicians and Egyptologists. A large body of students will, therefore, be grateful to the Trustees of the British Museum for placing in their hands the actual text of the papyrus in the form of a facsimile, which in beauty and accuracy of reproduction leaves nothing to be desired.

NEW ACCOUNT OF THE CHARACEÆ OF EUROPE.

Synopsis Characearum europearum. By Dr. Walter Migula. Pp. 176. Illustrated by woodcuts. (Leipzig 1898.)

THIS is an abridgment of the author's large and elaborate work on the subject, which forms vol. v. of the new edition of "Rabenhorst's Kryptogamen-Flora von Deutschland, Oesterreich und der Schweiz." The latter is by far the most extensive and minute account of the European Characeæ which has yet appeared; but the present work, containing as it does almost all the illustrations, and most of the letter-press information in a condensed form, will, we think, be found the more useful for all ordinary purposes.

The first eighteen pages are taken up with an account of the structure of the Characeæ, with illustrations, and a chapter on collecting, examining, and preserving specimens. These are followed by a key to the genera and species. At the end there is a list of books and memoirs dealing with the Characeæ, also one of published sets of specimens.

The remainder of the book consists of a tolerably complete description of the genera and species, with figures of nearly all the latter. The arrangement followed is almost entirely that of Braun's "Fragmente"; but the anomalous species *Lychnothamnus stelliger* of Braun is elevated to a separate genus, *Tolypellopsis*, Migula (*Chara*, sect. *Tolypellopsis*, Leonh.), and *Chara rudis*, *C. horrida*, *C. crassicaulis*, and *C. delicatula*, which Braun regarded as sub-species, are treated as species. The author has gone to extreme lengths in the differentiation of varieties. For instance, *C. aspera* is credited with thirty-nine named forms, *C. fragilis* with thirty-seven, *C. crinita*, *C. ceratophylla*, *C. intermedia* and *C. contraria* have each more than twenty; while *C. fœtida* (excluding *C. crassicaulis*) has actually sixty-nine! The majority of these forms are of the author's own describing. From the larger book we learn that many of them are based on plants from single localities, and, judging by the citations, it is fair to suppose in some instances from individual specimens. Considering the extreme variation to which the vegetative parts of these plants are subject, it seems to us a great pity to multiply names by distinguishing every trivial form.

In the nomenclature of the species Dr. Migula has implicitly followed Braun, in several instances quite disregarding the accepted rules. It would, we think, have been as well to have mentioned some of the more generally known synonyms.

The book is, unfortunately, entirely in German. As it deals with the Characeæ of the whole of Europe, it would have been more widely appreciated had the key and the diagnoses of the genera and species, at least, been in Latin.

As we have remarked, the illustrations are the same as those in the larger work. They are on the whole decidedly good, especially those of the magnified portions of the plants, and several of the species are more completely figured than in any earlier book. Some of the representations of the entire plants are somewhat crude and inartistic, and that of *Nitella translucens* is very poor;

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